Mathematics in Industry Study Group

South Africa

2021

Information Booklet

MISG

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Background

A Mathematics in Industry Study Group (MISG) is a five-day workshop at which academic researchers and graduate students work collaboratively with representatives from industry on research problems submitted by local industry. These Study Groups have been organized for over forty years in many countries around the world; the first Study Group in South Africa was held in 2004.

This year's Study Group will be held from Monday 1 to Friday 5 February 2021, and will be held virtually for the first time. The groups will be made up of graduate students attending and academics and applied mathematicians from South African Universities and invited international guests.

Design of the Study Group

- On the first day of the meeting, the industry representatives provide an outline of the problems posed by local industry.
- Applied mathematicians and graduate students, divided into small groups, then work collaboratively on the problems of their choice for three days. On the last day of the meeting, progress reports and/or presentations are presented back to the industry representatives.
- Further work is done on the problems after the event and a technical report for the proceedings, featuring outcomes, is produced for each problem later in the year.

Benefits of the Study Group

For industry

- Have leading South African and international mathematicians work on their proposed problem.
- Obtain an initial modelling of their problem and a development of the solution process. Hence, new perspectives and fresh ideas are brought to the problem.
- Establish research links with applied mathematicians which can develop far beyond the problem posed and establish contacts and develop employment opportunities for graduate students.

For academics

- Work on problems of genuine practical importance and perform good mathematics in the process and apply knowledge and skills to significant practical problems.
- Open up new research areas, leading to publications and new research collaborations.

For graduate students

- Learn to model and solve problems of genuine practical importance by working closely with leading academics and invited guests.
- Learn leadership skills by observing the way the moderator coordinates the work on the problem.
- Develop skills in presenting research material and skills in scientific communication.

Confidentiality and Intellectual Property

The Study Group is open and free from any non-disclosure agreement with industry. All information pertaining to both the problem statements in their initial form, the work done, and the outcomes of the problems is regarded as Open Access. There is only a voluntary payment by industry for submitting a problem, to encourage industry participation. This contribution will be used to run the Study Group and finance the participation of graduate students, academics, and invited guests.

Information for companies

- Most industrial problems can be modelled mathematically. Problems that Study Groups in South Africa have investigated include rock bursts, hydraulic fracturing, optimization in mining, traffic flow, flow of molten glass, HIV in the workplace, optimal collection of produce in rooftop farming in Soweto and South Johannesburg, renewable energy, wind turbines, robot motion, and image processing.
- Since the Study Group discuss the problems openly, the problems must not be of a confidential nature, and must be such that the results can be published.
- There is only a voluntary charge for submitting a problem, to encourage industry participation.
- Once a problem is selected, the industry representative needs to submit a detailed problem description well before the meeting and present the problem at the first day of the Study Group.
- The industry representative is expected to be present or contactable during the whole week to assist the academic participants.

Information for participants

- There is no registration fee, as participants give their expertise free of charge.
- The 2021 event will be held virtually via the Zoom platform, and support with be provided by the DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS).

Previous projects

Previous project 1 – Models for honeybee arrival and blossom phenology (2020)

Industry: Climate change

Problem Statement:

Phenology refers to the timing of annually recurrent biological events and their biotic and abiotic drivers. These phonological events are triggered by the change in seasons, as temperatures change, and rainfall begins or terminates. Under climate change, the timing of these events is shifting. In many instances, events such as blossoming that marked the beginning of spring are now occurring in late winter. Each species, however, responds to a discrete selection of abiotic forces. For one species, the trigger may be the occurrence of daytime temperatures warmer than 20 degrees Celsius; for another it may be the timing of first rainfall. This results in a progressive mismatch between pollinators and predators and prey, as one species will arrive/hatch/emerge from hibernation long before its food supply is available. In this problem we are considering the plight of honeybees under climate change, because of the progressive advance of blossoming in South Africa. Models for honeybee arrival and blossom phenology will need to be developed and compared to calculate the threat of mismatch.

Outcomes:

- Blossom phenology and the advancement in flowering dates was studied. A simple model for blossoms that is independent of bee behaviour was developed.
- Three phases in one blossoming cycle were considered: acceleration, saturation, and decay. The various stages in this cycle were accurately described using a beta distribution.
- An in-depth knowledge of the complicated biological processes which occur was not required to obtain a reasonable understanding of blossom behaviour.

- A compartment model was developed to model a single honeybee hive. Two sub-populations were identified: hive bees and forager bees. Hive bees are responsible for maintaining the hive whilst forager bees serve the hive by replenishing the food supply. Forager bees have a much higher death rate and are vulnerable to elimination.
- The effect of blossoms on bee numbers was incorporated through a system of equations describing the available food supply in the hive.
- The next step in this project is to now incorporate the effect of the advancement in phenology of bees and blossoms to predict the behaviour of honeybees in the future.

Previous project 2 – Green roofs to mitigate the urban heat island (2020)

Industry: Climate change

Problem Statement:

The Urban Heat Island effect, where a city is a few degrees hotter than the surrounding countryside, is a well-known problem. This is caused by the prevalence of dense and dark coloured materials. The dense materials, such as concrete, absorb solar radiation in the daytime and store it until the evening, when the heat is released into the atmosphere, as these materials absorb much more heat than lighter colours that reflect most of the solar radiation back into the atmosphere. This causes an up-swell of air that induces thunderstorms over the city, leaving the surroundings in a rain shadow.

City planners around the world are investigating strategies to add green spaces to help mitigate this effect. Roofs make up about 20-25% of the urban surface area and the conversion to green roofs is relatively inexpensive, hence conversion may be the most cost-effective way of controlling climate change in urban areas. Johannesburg inner city is built largely of reinforced concrete with plat roofs that are ideal for the installation of green roofs as no additional structural strengthening is required.

The problem is to model the existing Urban Heat Island in comparison with the natural terrain that would have characterised the pre-development state (taken as the temperature data from the Johannesburg Botanical Gardens). Various percentages of the area of the city with green roofs can then be modelled to determine an optimal area under green roof in the city.

Outcomes:

- It was determined that a clean concrete roof is better at reducing energy absorption than one painted green.
- The darker the roof the more heat absorption. However, the energy absorption is significantly reduced when a plant layer is introduced. It was found that after 4 hours the soil was some 4 degrees cooler than

the concrete. This resulted in a more-than 20% reduction in the energy stored in the soil when compared to that in the concrete. Taken over a whole city this reduction makes a huge difference.

- The reason that evapotranspiration is so important is that the solar energy is converted to latent heat (turning liquid water into vapour). This conversion of matter from one state to another is very energy intensive, hence the huge reductions in energy absorption.
- An important effect is that the plant layer shades the heat absorbing surface, preventing much absorption. There is also an air layer which can take the heat away rather than allow it to be stored for later release. Hence, we would expect even greater improvements by converting to green roofs.
- In future work we intend to improve the equation and correctly incorporate it into the heat flow model. We must also account for finite thickness layers.
- A sensible goal is to verify the work against experiments carried out at the University of the Witwatersrand, and, on a larger scale, by city councils.

Previous project 3 - Juice holdup detection in a sugar cane diffuser (2020)

Industry: Sugar cane processing

Problem statement:

To efficiently remove sugar from crushed and shredded cane in a sugar cane diffuser, it is necessary to maintain the water level within the cane bed. If the water level is too high, then flooding occurs, leading to shortcutting of the desired recycling of the flow within the diffuser. If, on the other hand, the water level is too low, then sugar will not be efficiently removed from the upper parts of the cane bed. Maintaining the water level at the correct level is, however, not easy, not least because the permeability of the cane bed is significantly non-uniform.

To facilitate the process of maintaining the derived water level within the cane bed, recessed viewing windows are incorporated into the side walls of the diffuser so that the operators can adjust the flow rates from the water sprays above the cane bed based on their observations of the water levels within the windows. It is not clear, however, if the water levels visible within the windows relate to those within the cane bed and, if they do relate, what the nature of this relationship is.

Outcomes:

- Initially a simple hydrostatic model with gravitational forcing was proposed. However, the results obtained from this model appeared to significantly overpredict the water level within the cane bed.
- A more detailed model describing the diffusion of water through the cane bed in the spirit of the classical work by Darcy and Richardson was also investigated.
- This latter model is challenging to solve, but it is hoped that it may still be possible to obtain useful results from one or more appropriately simplified versions of the model.

Previous project 4 - Optimising Urban and Rooftop Farming in Soweto and South Johannesburg (2019)

Industry: Agriculture

Problem Statement:

Urban and roof-top farming is important for food security in South Africa, and entrepreneurial and employment opportunities in urban farming have risen significantly in Soweto and South Johannesburg.

In contrast to traditional farming, urban and roof-top farming yields small crop sizes and are spread out throughout the city, and roof-top farming can be done year-round as it is farmed hydroponically under controlled conditions. Hydroponic farming requires no soil; instead, the roots are suspended in mineral-rich static water, or continuously moving water, while ultraviolet lamps provide continuous artificial sunlight. On roof-tops, plants are grown in greenhouses to protect them from extreme temperatures, wind and pests. Farms can now produce food throughout the year.

The produce of the farms must be collected and transported to the market. With urban farming, there are many smaller farms in contrast to the norm of one or two large farms, and these smaller farms are also separated by some distance.

Since the farming can be done throughout the year, the Study Group was asked to optimise the plant and harvest times for many farms producing a range of products. Prices at different times of the year in South Africa and growing times for the produce were provided.

The Study Group was also asked to optimise the routes, by road, between the farms during the collection of produce. Some produce requires refrigeration, which not all vehicles have. The optimal routes for non-refrigerated vehicles therefore differ from that for refrigerated vehicles. The locations of the farms were provided. The Study Group had to determine how the routes were calculated, for example, with Google maps.

Outcomes:

- The model allows for the coordinated logistics and transportation of farm produce.
- The results can be used as a benchmark for the mathematical model formulation problem.
- The main points to improve are:
  - formulating and finding an effective solution of the predictive price model for planting and harvesting times,
  - find efficient solution of the mathematical model for routing planning,
  - possibly combine the two models which were developed to have a better coordinated logistics system among the farms.

Previous project 5 – Lake Kivu Surface Water Pollution (2019)

Industry: Energy

Problem Statement:

Lake Kivu in East Africa is known to be a dangerous lake, and a new type of hazard has recently been taken under consideration regarding its relationship with the existence of high concentration of dissolved gas in the water at depth in the lake. This new hazard is water pollution, and it seems that living around Lake Kivu means accepting a higher risk than living elsewhere in the region.

The most obvious type of water pollution affects surface water, and most water pollution does not begin in the water itself. The literature argues that there is a quantity of gas vented in the atmosphere during the harvest and there are other toxic substances entering the lake; all those substances get dissolved or lie suspended in water, and chemicals released by smokestacks (chimneys) can enter the atmosphere and then fall back to earth as rain, entering the lake and causing water pollution. The pollution may cause the quality of the water to deteriorate, and further affects the aquatic ecosystems. These pollutants can also seep down and affect the groundwater deposits or disturb density-stratified lake waters.

In general, water pollution has many different causes, and this is one of the reasons why it is such an interesting problem to solve. There is a clear danger lying ahead, and something needs to be done in advance to alert decision-makers from all points of view; environmentally, and economically.

Methane extraction by venting a certain percentage of gas in the atmosphere would increase the risk of water pollution, which is why the continuous monitoring and regulation is mandatory. Therefore, the extraction projects must be regulated and well monitored. The Study Group were asked to model the surface water pollution in Lake Kivu.

Outcomes:

- The released gases and particles end up in the lake either during or directly after rain.
- These gases and particles can affect the health of inhabitants, animals and water life.
- This project illustrates an approach that may be used to obtain crude estimates for the severity of the pollution problem.

Previous project 6 – Algorithm to count modern houses from lidar data sets over rural areas in Mpumalanga (2018)

Industry: Environmental monitoring

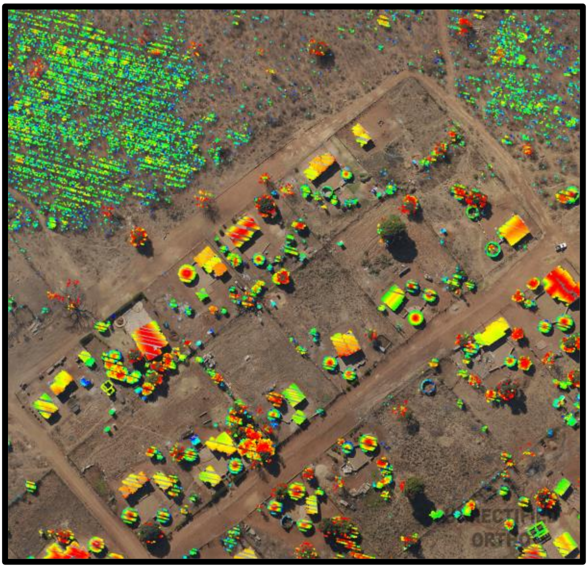
Problem statement:

Air-born Light image Detection And Ranging (LiDAR) systems provide a high resolution, 3-D picture of any landscape. The picture is generated by tracking the time-to-return of individual laser pulses that bounce off objects on the ground. The time-to-return is used to measure height and combined with highly accurate GPS systems, generates a 3-D point cloud, with each point marking where a laser pulse hit an object.

Modern lidar systems create point clouds with up to 20 points per square meter, over potentially thousands of hectares, resulting in very large data sets of x-y-z data. Off-the-shelf processing tools are very good at extracting features of interest (e.g. power lines, trees, roads, roofs etc.) from this data set, but these are typically designed for European urban environments. Through a large research collaboration, we have acquired extensive lidar data sets over rural areas in Mpumalanga, and we need an algorithm to count modern houses. By modern, we refer to brick structures, with angled and tiled roofs, constructed with pre-fab wooden trusses. By counting the change in the number of such tiled roofs, we could develop a measure of economic welfare that covers much larger areas than would be possible through conventional survey techniques. The challenge is to extract a clustered collection of connected, small angled planar surfaces, that would represent the roof of a new house, from a large 3D point cloud of millions of points.

Outcomes:

- The Hough transform offers a reliable measure of the number of planes in the candidate house, thus allowing machine classification of all the houses in the image.
- In addition to implementing the 3-D Hough transform, an additional multi-model fitting algorithm should be investigated.
- A significant amount of coding and testing needs to be done to create an efficient software system.
- A convolutional neural network might be able to learn the signature of a modern house directly, from a fairly large set of LiDAR training data. If this works, it could lead to an extremely efficient counting algorithm.



Previous project 7 – Spontaneous combustion of stock-piled coal (2018)

Industry: Coal mining

Problem statement:

Spontaneous combustion of stock-piled coal is a well-known problem in the coal mining industry. The effects of individual causal factors are known, but typically for a specific location, or conditions at a location.

The interaction between causal effects, and how their relative importance varies with environmental conditions, are not well understood. The height and shape of a stockpile, variation in grain size, degree of compaction, oxygen availability, moisture content, and chemical composition of the coal are all known contributing factors to the probability of spontaneous combustion in coal.

Outcomes:

- Pile size was found to be a strong contributing factor.
- For accuracy of the model, the internal consumption and transport of oxygen must be taken into account.
- These models form a basis for future investigations using sophisticated models which could, for example, explore the influence of stockpile geometry.

MISG 2021 Organising Committee

Professor David Mason, University of the Witwatersrand, Johannesburg

Prof Mason is an Emeritus Research Professor in the School of Computer Science and Applied Mathematics at the University of Witwatersrand, Johannesburg. He has conducted pioneering work in the modelling of problems in solid and fluid mechanics and the use of mathematical techniques to derive analytical solutions. As the founder of MISG South Africa, he has overseen the event since 2004. **Email:** David.Mason@wits.ac.za

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Dr Krishnannair is a senior Lecturer in the Department of Mathematical Sciences at the University of Zululand, and she holds a PhD from Stellenbosch University. Her research focuses on the development of advanced multiscale and multivariate techniques for the analysis of complex high-dimensional datasets such as time series data from chemical processes. Her current major research is on the development of an embedded artificial intelligence in mining. **Email:** KrishnannairS@unizulu.ac.za

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Prof Ali holds a chair the School of Computer Science and Applied Mathematics at University of the Witwatersrand, Johannesburg, and has a PhD from Loughborough University, UK. He has worked as a postdoctoral fellow at Plymouth University, UK, and Abo Akademi, Finland. His research areas of interest include stochastic methods for global optimization, programming problems, and industrial mathematics. **Email:** Montaz.Ali@wits.ac.za

Dr Naeemah Modhien, University of the Witwatersrand, Johannesburg

Dr Modhien is a lecturer in the School of Computer Science and Applied Mathematics at the University of Witwatersrand, Johannesburg. In 2017 she obtained her PhD from Wits University, entitled "The effect of suction and blowing on the spreading of a thin fluid film: a Lie point symmetry analysis". Her research interests are fluid mechanics, Lie point symmetries, and partial differential equations. **Email:** Naeemah.Modhien@wits.ac.za

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Prof Khalique received his PhD from the University of Dundee, UK, and he is currently a full professor in the Department of Mathematical Sciences at North West University. His research interests are in Lie group analysis of differential equations. He is a Fellow of the Royal Society of South Africa and the Institute of Mathematics and its Applications, UK, and member of the Academy of Science of South Africa, London Mathematical Society, South African Mathematical Society, and Society for Industrial and Applied Mathematics, USA. **Email:** Masood.Khalique@nwu.ac.za

Mr Erick Mubai, University of the Witwatersrand, Johannesburg

Mr Mubai holds a MSc degree in Computational and Applied Mathematics and is currently pursuing a PhD at the University of the Witwatersrand. The focus for his dissertation was on using the associated Lie point symmetries to reduce the governing coupled partial differential equations of a thermal jet to ordinary differential equations. His current research area is fluid flows such as jets and wakes, and how they affect the temperature field. **Email:** Erick.Mubai@wits.ac.za

Ms Diane Dowejko, DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences

Ms Dowejko is the Research Administrator at the DSI-NRF Centre of Excellence in Mathematical and Statistical Science. She obtained her BA (Hons) in Language and Media from the University of Brighton, UK, and holds a DELTA from the University of Cambridge. She is currently undertaking an MA in linguistics at Rhodes University, and her research interests lie in Legitimation Code Theory and psycholinguistics. **Email:** diane.dowejko@wits.ac.za

Professor Jeff Sanders, African Institute for Mathematical Sciences

Prof Sanders has been at AIMS South Africa since early 2012, where he is currently Senior Resident Researcher. His interests lie in Abstract Harmonic Analysis and Theoretical Computer Science, and at MISG he promotes the solution of IT-related problems using discrete mathematics and distributed algorithms. Jeff was born and educated in Australia, but spent much of his career in Oxford where he enjoyed productive industrial research with IBM and inmos. **Email:** jsanders@aims.ac.za

Dr Simukai Utete, African Institute for Mathematical Sciences

Dr Simukai Utete is the Academic Director of AIMS-South Africa. She led the Mobile Intelligent Autonomous Systems (MIAS) field robotics group at the Council for Scientific and Industrial Research (CSIR) in Pretoria for over 4 years. She holds a DPhil in Robotics (Engineering Science) from the University from Oxford University. Her research interests include data fusion and multi-robot systems. **Email:** simukai@aims.ac.za

Dr Adewunmi Fareo, University of the Witwatersrand, Johannesburg

Dr Fareo has a PhD in Applied Mathematics from the University of the Witwatersrand, Johannesburg, where he has been lecturing since 2012 in the school of Computational and Applied Mathematics. He has published various works in the field of fluid mechanics, and solid mechanics and modelling. **Email:** Adewunmi.Fareo@wits.ac.za

MISG 2021 Invited Guests

Professor Neville Fowkes, University of Western Australia, Australia

Prof Fowkes, a senior lecturer at the University of Western Australia, is a mathematical modeller who works mainly on continuum mechanics problems arising out of industrial contexts. He has participated in more than 60 Mathematics in Industry Study Groups held in different countries and has authored a text on mathematical modelling based largely on problems arising out of these experiences. His research interest areas include crack propagation and groundwater flow. **Email:** Neville.Fowkes@uwa.edu.au

Professor Tim Myers, Centre de Recerca Matematic, Barcelona, Spain

Prof Myers is currently the Head of the Industrial Mathematics group at the Centre de Recherches Mathématiques, and an Adjunct Professor at the Universitat Politècnica de Catalunya. His research involves building and analysing models of physical problems, and one of his focus areas is nanotechnology, where he works on problems in nanofluid heat transfer, melting of nanoparticles and Ostwald ripening of nanoparticles. **Email:** tmyers@crm.cat

Professors Hilary and John Ockendon, Oxford University

Hilary and John Ockendon are both Emeritus academics at Oxford. Their careers have focussed on real-world problems that can only be well-understood by using mathematics. They believe in collaborative research and have benefitted from the mathematical impetus provided by the many Study Groups in which they have participated around the world since the first one in Oxford in 1968. This has stimulated them to write many papers and books across applied mathematics. **Email:** ock@maths.ox.ac.uk

Professor Ian Griffiths, Oxford University

Ian Griffiths is Professor of Industrial Mathematics at the University of Oxford's Mathematical Institute. His principal research agenda is in industrial fluid dynamics. He has worked with Dyson to improve vacuum cleaner performance, with Schott to develop a method for manufacturing glass which is now used in the manufacture of smartphones and bendable displays, and most recently with Smart Separations on the development of a novel household air-purification device that annihilates coronavirus. **Email:** ian.griffiths@maths.ox.ac.uk

Dr Shariffah Suhaila Syed Jamaludin, Universiti Teknologi Malaysia

Dr Shariffah Suhaila Syed Jamaludin is a senior lecturer in the Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia. Currently, she is one of the research fellows in the UTM Centre for Industrial and Applied Mathematics (UTM-CIAM). Her research interests include Statistical Modelling and Analysis for Climate and Hydrological Data, Functional Data Analysis, Spatial Data Analysis, and Disease Modelling using Generalized Linear and Additive Models. **Email:** suhailasj@utm.my

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